ISSN: 1545-679X
Information Systems Education Journal
Volume 6, Number 20 http://isedj.org/6/20/ February 22, 2008

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Two Gestalts for Mathematics: Logical vs. Computational

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Abstract: Many educators have campaigned to increase mathematical content in the curriculum guidelines for Information Systems. Unfortunately, mathematical concepts are often presented in a manner that conflicts with the general mental framework, or gestalt, of most IS students. But fortunately there is more than one gestalt in mathematics. This paper attempts to characterize and measure two gestalts for mathematics–one based on proving theorems and the other based on solving problems. Our methodology assumes that words used frequently in a textbook indicate the gestalt of the author. By comparing word frequencies in various mathematics books, we developed Logical Math and Computational Math scales for measuring, respectively, the theorem-proving and problem-solving gestalts of the authors. We examine the concepts that form the core of each scale, and highlight the areas of mathematics that score high on each scale. Our findings have relevance in the development of approaches for teaching mathematical topics in computing courses.

Keywords: gestalt, mathematics, scale, computational, logical, IS education

Recommended Citation: McMaster, Rague, McMaster, and Blake (2008). Two Gestalts for Mathematics: Logical vs. Computational. *Information Systems Education Journal, 6* (20). http://isedj.org/6/20/. ISSN: 1545-679X. (Also appears in *The Proceedings of ISECON 2007:* §2342. ISSN: 1542-7382.)

This issue is on the Internet at **http://isedj.org/6/20/**

The **Information Systems Education Journal** (ISEDJ) is a peer-reviewed academic journal published by the Education Special Interest Group (EDSIG) of the Association of Information Technology Professionals (AITP, Chicago, Illinois). • ISSN: 1545-679X. • First issue: 8 Sep 2003. • Title: Information Systems Education Journal. Variants: IS Education Journal; ISEDJ. • Physical format: online. • Publishing frequency: irregular; as each article is approved, it is published immediately and constitutes a complete separate issue of the current volume. • Single issue price: free. • Subscription address: subscribe@isedj.org. • Subscription price: free. • Electronic access: http://isedj.org/ • Contact person: Don Colton (editor@isedj.org)

This paper was selected for inclusion in the journal as part of the ISECON 2007 best papers group. Best papers received preliminary reviews by three or more peers placing them in the top 30% of papers submitted and final reviews placing them in the top 15% by three or more former best papers authors who did not submit a paper in 2007.

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Two Gestalts for Mathematics: Logical vs Computational

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ABSTRACT

Many educators have campaigned to increase mathematical content in the curriculum guidelines for Information Systems. Unfortunately, mathematical concepts are often presented in a manner that conflicts with the general mental framework, or gestalt, of most IS students. But fortunately there is more than one gestalt in mathematics. This paper attempts to characterize and measure two gestalts for mathematics--one based on proving theorems and the other based on solving problems. Our methodology assumes that words used frequently in a textbook indicate the gestalt of the author. By comparing word frequencies in various mathematics books, we developed Logical Math and Computational Math scales for measuring, respectively, the theorem-proving and problem-solving gestalts of the authors. We examine the concepts that form the core of each scale, and highlight the areas of mathematics that score high on each scale. Our findings have relevance in the development of approaches for teaching mathematical topics in computing courses.

Keywords: gestalt, mathematics, scale, computational, logical, IS education

1. INTRODUCTION

In recent years, many educators have campaigned to increase mathematical content in the curriculum guidelines for Information Systems and Computer Science (ACM 2001). Reasons for this zeal include faith in the positive effects of mathematics on the mind (Ralston, 2005), as well as the belief that mathematical logic skills improve the software development process (Bruce, 2003).

If one agrees with this position, then one must deal with how mathematical topics should be presented to IS students. In an ideal world, the mathematical ideas would blend naturally into the general mental framework, or *gestalt*, of these students.

In *The Mathematical Experience*, Davis and Hersh (1981) state:

> People vary dramatically in what might be called their *cognitive style*, that is, their primary mode of thinking.

Ken Bain (2004) adds:

> The students bring paradigms to the class that shape how they construct meaning. Even if they know nothing about our subjects, they still use an

> existing mental model of something to build their knowledge of what we tell them.

Unfortunately, the gestalt of mathematics often conflicts with the cognitive style of most IS students. But fortunately there is more than one gestalt in mathematics. In his classic book *How to Solve It*, Polya (1945) describes two faces of mathematics:

> Studying the methods of solving problems, we perceive another face of mathematics. Yes, mathematics has two faces; it is the rigorous science of Euclid, but it is also something else. Mathematics presented in the Euclidean way appears as a systematic, deductive science; but mathematics in the making appears as an experimental, inductive science. Both aspects are as old as the science of mathematics itself.

Fifty years later, Velleman (1994) wrote a book called *How to Prove It*, in which he discusses the same two faces of mathematics but in reverse order:

> This textbook will prepare students to make the transition from solving problems to proving theorems by teaching them the techniques needed to read and write proofs.

This paper presents a preliminary attempt to characterize and measure two gestalts for mathematics--one based on *proving theorems* and the other based on *solving problems*. Measurement of mental concepts is always difficult. Our methodology is based on the assumption that the words people use are suggestive of their mental state. In particular, the words used frequently in a textbook indicate the gestalt of the author.

By comparing word frequencies in various mathematics books, we have developed two scales for measuring the mathematical gestalt of the authors. A Logical Math scale measures theorem-proving gestalt, and a Computational Math scale measures problem-solving gestalt. We use the term "computational" in the broad sense described by Wing (2006) in her paper on "Computational Thinking".

Later in this paper, we apply our gestalt scales to the books used to develop the scales. Our purpose is to determine the nature of each type of gestalt, and which mathematical areas tend to have high scores on a particular scale. We use these results to infer where proving theorems predominates in mathematics, and where solving problems is the preferred framework. Our findings have relevance in the development of approaches for teaching mathematical topics in computing courses.

2. CURRENT APPROACHES

Before constructing measures for our two gestalts, we searched the literature for terminology and descriptions of related approaches to mathematics. For theorem-proving gestalt, commonly used synonyms include pure, formal, logical, symbolic, deductive, dialectic, analytic, and rigorous. Words representing problem-solving gestalt include applied, realistic, inductive, constructive, computational, algorithmic, analog, and intuitive. These words often appear in diametric pairs (e.g. pure vs. applied, deductive vs. inductive, rigorous vs. intuitive). Three framework pairs of special interest are summarized below.

Dialectic vs. Algorithmic Mathematics

Henrici (1974) describes the difference between *dialectic* and *algorithmic* mathematics as follows:

> Dialectic mathematics is a rigorously logical science, where statements are true or false, and where objects with specified properties either do or do not exist. Algorithmic mathematics is a tool for solving problems. ... Dialectic mathematics invites contemplation. Algorithmic mathematics invites action. Dialectic mathematics generates insight. Algorithmic mathematics generates results.

Davis and Hersch (1981) remark that changing from one of these *paradigms* to the other can be discomforting:

> There is a distinct paradigm shift that distinguishes the algorithmic from the dialectic. ... People who have worked in one mode may very well feel that solutions within the second mode are not "fair" or not "allowed."

Body vs. Soul

Body and Soul is a program in mathematics education reform at Chalmers University of Technology in Sweden. Contributors to the program have written books and software for teaching applied mathematics with a blend of *computational* (body) and *analytical* (soul) elements. In the Calculus and Linear Algebra textbook (3 volumes), Eriksson (2003) writes:

> It would of course be ideal to combine the more rigorous aspects of mathematics with the problem-solving aspects.... This may sound very difficult, but it is exactly what the three volume textbook *Applied Mathematics: Body and Soul* ... aims to do: to combine the constructive /computational (body) aspects with the symbolic (soul) aspects of mathematics in undergraduate teaching.

Horizontal vs. Vertical

Realistic Math Education (RME) is a Dutch reform movement for mathematics education. It is based on the work of Treffers (1987), who encourages two types of "mathematization", *horizontal* and *vertical*:

> In horizontal mathematization, the students come up with mathematical tools which can help to organize and solve a problem located in a real-life situation. Vertical mathematization is the process of reorganization within the mathematical system itself.

Freudenthal (1991) restated the RME approach as:

> Horizontal mathematization involves going from the world of life into the world of symbols, while vertical mathematization means moving within the world of symbols.

This is in contrast to the "theorem before application" tendency in traditional mathematics courses. Freudenthal stressed that these two forms of mathematization are of equal value.

Dialectic mathematics and analysis (soul) reflect theorem-proving gestalt. Algorithmic mathematics and computation (body) are concerned with solving problems. For the third framework pair, the division is not as simple. In horizontal mathematization, students start with a real-world problem and develop a mathematical model. This is a problem-solving activity. Vertical mathematization requires students to work with the model until they uncover one or more algorithms. Experimentation to find an algorithm is a problem-solving activity. Verifying the correctness of an algorithm leads to theorem proving.

Both gestalts encourage abstraction through manipulating symbolic objects and constructing models. Ultimately, it is the instructor who must choose an appropriate blend of theorem-proving gestalt and problem-solving gestalt in computing courses. But what do these mathematical gestalts consist of? How can course concepts be integrated into these mental frameworks? To answer these questions, we devised a methodology for measuring these gestalts and identifying their essential features.

3. METHODOLOGY

The methodology used to develop measuring instruments for our two gestalts is described in this section. We constructed a Logical Math scale for theorem-proving gestalt and a Computational Math scale for problem-solving gestalt. The methodology involved the following steps:

1. Choose a broad sample of mathematics books.
2. Record frequencies for words used most often in the books.
3. Convert nouns, verbs, adjectives, and adverbs to a consistent form.
4. Transform the frequencies to make data from different books comparable.
5. Combine synonyms into "word groups".
6. Construct the gestalt scales.

Sampling

By design, a wide variety of mathematics books were sought for our sample. We did not want one mathematical area to unduly influence our measures of gestalt. Books were chosen from traditional mathematics fields such as Algebra, Analysis, Geometry, Number Theory, and Topology, together with applied fields such as Numerical Analysis, Probability, Statistics, and Operations Research. Our list of applied math topics was influenced by programs in Computational Mathematics at schools such as Princeton University, California Institute of Technology, and University of Waterloo.

We selected books from the Amazon web site that included a *concordance* (a list of frequently used words). Our need for a concordance limited our choice of books. Amazon does provide a concordance for many of its books, so we were able to get a diverse sample. The majority of our sample books are suitable to use as college textbooks, but some are aimed at different markets.

At the time this paper was written, our sample consisted of 112 mathematics books. We made an *a priori* classification of each book as either Traditional Math or Applied Math.

This classification was based on words used in the book title (e.g. "theory", "applied", "computational"), on book reviews, and on the area of mathematics covered by the book. For example, a book with the title "Theory of Numbers" would be classified as Traditional Math, whereas a book with the title "Computational Differential Equations" would be classified as Applied Math. The initial classification resulted in 56 Traditional Math books and 56 Applied Math books.

Data Collection

The Amazon concordance for a book provides a list of the 100 most frequently used words. The Amazon concordances screen out many (but not all) common English words, such as "the" and "of". For each concordance word, we recorded the book code, word, and frequency (FREQ). Frequency is the actual number of times a word occurs in a book.

Convert Words to a Consistent Form

One problem with using words to build scales is that words can take more than one form. For example, nouns may be singular or plural. To alleviate this problem, we converted many words to a consistent form. We did not want the scale contribution of a word to depend on the particular form or tense an author favored. The following types of word conversions were performed:

a. Convert plural nouns to singular form ("elements" becomes "element").

b. Make verbs refer to plural subjects ("exists" becomes "exist").
c. Change verbs to present tense ("defined" becomes "define", "solving" becomes "solve").
d. Remove endings such as "al" and "ly" from some adjectives and adverbs ("computational" becomes "computation", "finitely" becomes "finite")

Transform Frequencies

Word frequencies needed to be rescaled (or standardized) because books vary in their total number of words. We rescaled word frequencies within a concordance as follows:

a. We removed all words that are in the list of Top 100 Common English Words (Fry et al, 1993). Fortunately, Amazon had already removed most of these Top 100 words. Otherwise, we would have had few words left to analyze.
b. For the remaining (approx. 90) words, we calculated the average frequency (AvgFREQ).
c. We the restated each word frequency relative to the average frequency using the formula:

```
StdFREQ = (FREQ/AvgFREQ)*100
```

With this calculation, a standard frequency (StdFREQ) score of 100 represents the transformed frequency for the "average word" in the reduced concordance. A word with a StdFREQ value of 300 would appear three times as often as the average concordance word in the same book.

We did not transform the frequencies to compensate for Zipf's law. Zipf's pattern of skewness was diminished because many common English words had been removed from a book's list of words. Also, we used average standard frequency scores across books to develop the scales. This allowed central limit theorem effects on averages to further reduce the skewness.

Combine Synonyms into Word Groups

Another problem in using words to build scales is that different words can have the same meaning. When relevant, we combined two or more synonyms into a concatenated "word group". For example, "function" and "map" become "function/map". We applied this step after standardizing word frequencies (StdFREQ) because we wanted the average frequency for a concordance to be based on individual words. When synonyms were later combined into word groups, the StdFREQ score for the group was the sum of the StdFREQ scores of the words in the group.

Some concordance words were mathematical abbreviations (e.g. "lim" as an abbreviation for "limit"). In all of these cases, the frequencies were low or confined to a small number of books, so combining the words did not lead to the inclusion of the word group on a scale.

Construct Gestalt Scales

Constructing the Logical Math scale (referred to as LMATH) and the Computational Math scale (referred to as CMATH) was an iterative process. In defining the gestalt scales, we looked for words that are used *frequently* within each book, and *consistently* across similar books.

First Iteration

1. Query all Traditional Math books for the LMATH scale. Query all Applied Math books for the CMATH scale. Find all words in which the average StdFREQ (AvgStdFREQ), taken across all books for that scale, is above a predefined value (e.g. AvgStdFREQ > 125 for the first iteration). Select only those words found in a predefined percent of the scale books (e.g. at least 60% for the first iteration). The choice of minimums for AvgStdFREQ and percent of books is subjective. Order the words by decreasing AvgStdFREQ. Because our sample was diverse and some books may have been misclassified, we set our cutoff criteria low for the first iteration and raised the levels in later iterations.
2. Subtract 100 from the AvgStdFREQ for each word selected in Step 1. Then sum these differences. For each word, we consider only the amount its AvgStdFREQ exceeds the frequency for an average concordance word.
3. The Weight for a scale word is AvgStdFREQ - 100, restated as a percent of all weights.

   ```
   Weight = 100*(AvgStdFREQ - 100)
             /(sum of differences)
   ```

   The sum of the weights over all words used in the scale is 100%.
4. Calculate the LMATH and CMATH scores for all 112 books in the sample. The score for an individual book is:

   ```
   LMATH (CMATH) scale =
       Sum[(Weight/100) * StdFREQ]
   ```

   where the sum is across all words used in the scale. The LMATH (CMATH) score for a book is a weighted average of the StdFREQ values for all scale words.

Additional Iterations (repeat as necessary):

1. Check the LMATH and CMATH scores for each book to see if any books are misclassified (Traditional vs. Applied). If so, reclassify them.
2. Remove books from the sample that have a relatively low score on their relevant scale.
3. Repeat the steps from Iteration 1 with the remaining books to obtain a revised list of words and weights for each scale, plus a new set of LMATH/CMATH scores for each book. Note that the StdFREQ values do not change from iteration to iteration.

In this study, we started with 56 Traditional Math books and 56 Applied Math books. After the first iteration, we determined that four books had been misclassified (higher on the "wrong" scale). We continued our iterations until the scale words and their rank order did not change. The main decision variables at each stage were the cutoff criteria for words (average StdFREQ and percent of books) and the choice of books used to determine the weights. We chose books with the highest LMATH or CMATH scores, which reinforced the scale words and weights in the next iteration. After three iterations and no further misclassifications, we had LMATH and CMATH scales constructed from 25 Traditional Math books and 25 Applied Math books.

4. DATA ANALYSIS

Using the methodology outlined in the previous section, two gestalt scales for mathematics were developed--a Logical Math scale and a Computational Math scale. Each gestalt scale consists of a list of words/groups and weights.

Logical Math Gestalt

The Logical Math (LMATH) scale consists of 10 words/groups and weights. The details of this scale are presented in Table 1.

The most frequent word group for the LMATH scale is "theorem/lemma /proposition", with an AvgStdFREQ score of 439.5. As expected, the concepts of "theorem", "proof", and "definition" (with equivalent words combined) are represented on the scale. Surprisingly, the most frequent individual word is "let". Words like "hence/thus/therefore", "show", "follow", and "since" are stylistic conventions commonly used to express logical ordering in proofs.

Word/Group	Books	Avg StdFREQ	Weight
theorem/lemma /proposition	25	439.5	19.12
let	25	416.2	17.81
proof/prove	25	341.2	13.58
function/map	25	315.1	12.11
set	25	281.3	10.21
hence/thus /therefore	25	235.9	7.65
definition /define	25	212.4	6.33
show	24	194.5	5.32
follow	24	174.2	4.18
since	24	165.5	3.69
TOTAL			100.00

Table 1: Logical Math Scale
Based on 25 Traditional Math books

The words "function/map" and "set" are on the scale because these terms are commonly used throughout mathematics. Words specific to only a few areas of mathematics (e.g. "derivative" and "ring") do not appear on the scale. Thus, LMATH measures a general gestalt for mathematics, as intended.

All scale words are used consistently in Traditional Math books. Each word appears in at least 24 of the 25 Traditional Math books used to construct the final LMATH scale. The AvgStdFREQ values for "theorem/lemma/propositon" and "let" are above 400, indicating that these words appear about four times more often than an average concordance word in Traditional Math books.

Our cutoff point in for including words in the final iteration of the LMATH scale was an AvgStdFREQ value of 150. We chose this value so that less meaningful words (e.g. "example") would not be included on the scale. If the cutoff point had been 200, then "show", "follow", and "since" would not appear on the scale. The StdFREQ values of the remaining words would not change, but the Weights would be different.

Computational Math Gestalt

The Computational Math (CMATH) scale consists of 9 words/groups and weights. The details of this scale are presented in Table 2.

The most frequent word for the CMATH scale is "problem", with an AvgStdFREQ score of 389.1. Word group "solution/solve" is the third most frequent item on the scale. This confirms that problem solving is a central theme in the Computational Math gestalt of Applied Math books.

Word/Group	Books	Avg StdFREQ	Weight
problem	25	389.1	19.28
method /algorithm	24	346.0	16.40
solution /solve	25	314.3	14.29
value /variable	24	267.1	11.14
equation	20	265.2	11.02
function/map	24	263.4	10.90
model	20	223.5	8.24
system	23	167.2	4.48
condition /constraint	25	163.8	4.25
TOTAL			100.00

Table 2: Computational Math Scale
Based on 25 Applied Math books

Other words on this scale go beyond Polya's procedures for solving mathematical problems. Gestalt as measured by the CMATH scale is concerned with how to use mathematics to solve real world problems. The words "model" and "method /algorithm" describe the main approach to solving problems, and words like "value /variable", "equation", "function/map", and "condition/constraint" are components of mathematical models and algorithms.

The word group "function/map" appears on both the LMATH and the CMATH scales, but with different Weights. We considered excluding these words from both scales to make the scales more "orthogonal". We decided instead to retain "function/map" on both scales and allow the two gestalts to overlap.

The CMATH scale words appear less consistently in Applied Math books than do the LMATH words in Traditional Math books. Most CMATH words appear in at least 23 of the 25 Traditional Math books used to build the scale, but "model" and "equation" appear in only 20 books. No word on this scale has an average StdFREQ score above 400, and the average StdFREQ score for "model" is only 223.5.

We feel that "model" is an essential part of Computational Math gestalt. According to Kramer (2007), "modeling is the most important engineering technique." Ideally, this word would appear frequently in nearly all Applied Math books. On the other hand, "model" almost never appears in Traditional

Math books. From the eyes of Logical Math gestalt, the real world is irrelevant, so there is no need for models.

We used the same cutoff point of 150 for including words in the final iteration of the CMATH scale. If during the construction of the CMATH scale, the minimum AvgStdFREQ value had been 200, then "system" and "condition/constraint" would be excluded from the scale. The scale would consist of the remaining words, with revised Weights.

Logical Math Example

We calculated Logical Math and Computational Math scores for all 112 books in the original sample. We can learn a lot about a book from the calculation of its LMATH and CMATH scores. The calculations show which words contribute most to the gestalt scores for a book:

(1) The StdFREQ value for a scale word indicates how often the word is used in the book.

(2) The Weight of a scale word defines the importance of the word for measuring gestalt.

The book receiving the highest LMATH score (394.2) was Bloch's *Proof and Fundamentals: A First Course in Abstract Mathematics* (2000). The LMATH calculations for this Traditional Math book are shown in Table 3.

Word/Group	Weight	StdFREQ	LMATH Scale
theorem/lemma /proposition	19.12	355.2	67.9
let	17.81	557.1	99.2
proof/prove	13.58	476.1	64.7
function/map	12.11	358.8	43.5
set	10.21	532.1	54.3
hence/thus /therefore	7.65	231.9	17.7
definition /define	6.33	298.8	18.9
show	5.32	219.6	11.7
follow	4.18	254.7	10.6
since	3.69	153.7	5.7
TOTAL			**394.2**

Table 3: LMATH Scale Values
Bloch -- Proofs and Fundamentals

Bloch's book includes all of the LMATH scale words. For Bloch, the most frequent words are "let" and "set", both with StdFREQ values above 530. All words from "theorem/lemma/proposition" through "set" have a StdFREQ value above 350. The LMATH value of 394.2 can be interpreted as follows: the weighted mix of scale words appears about 4 times more often than an average concordance word in this book.

Computational Math Examples

The book receiving the highest CMATH score (390.0) was Pardalos' *Handbook of Applied Optimization* (2002). The CMATH calculations for this Applied Math book are shown in Table 4.

Word/Group	Weight	StdFREQ	CMATH Scale
problem	19.28	631.0	121.7
method /algorithm	16.40	633.3	103.9
solution/solve	14.29	475.4	67.9
value/variable	11.14	221.0	32.4
equation	11.02	67.4	7.4
function/map	10.90	291.0	24.1
model	8.24	200.2	16.5
system	4.48	151.8	6.8
condition /constraint	4.25	218.5	9.3
TOTAL			**390.0**

Table 4: CMATH Scale Values
Pardalos -- Handbook of Applied Optimization (HAP)

Pardalos' book includes all of the CMATH scale words. For Pardalos, the most frequent words are "problem", "method/algorithm", and "solution/solve", each having a StdFREQ value above 475.

By comparison, Polya's *How to Solve It*, which provided the initial inspiration for this paper, has a CMATH score of 265.7. Polya uses the words "problem" (StdFREQ = 1005.3) and "solution/solve" (StdFREQ = 402.0) very heavily, but his approach to solving problems specifies neither "models" nor "algorithms". It appears that Polya is concerned with how to solve mathematical problems, rather than real world problems. The calculation of Polya's CMATH score is shown in Table 5:

Word/Group	Weight	StdFREQ	CMATH Scale
problem	19.28	1005.3	193.8
solution/solve	14.29	402.0	57.4
equation	11.02	57.9	6.4
condition /constraint	4.25	189.8	8.1
TOTAL			**265.7**

Table 5: CMATH Scale Values
Polya -- How to Solve It

The relationship between LMATH and CMATH scores for all Traditional Math and Applied

Math books in the sample is shown as a scatter diagram in Appendix 1. The top 25 Traditional Math books used to construct the LMATH scale (Top Traditional) and the top 25 Applied Math books used to construct the CMATH scale (Top Applied) are displayed with larger symbols in the graph. The remaining Traditional Math and Applied Math books are termed "Low" in the figure.

In this diagram, the LMATH and CMATH scores demonstrate how widely word usage varies both between and within the Traditional Math and Applied Math groups. This pattern of variation is consistent with the claim that more than one type of gestalt is involved in the process of preparing and writing Math textbooks, although one gestalt usually predominates.

5. MATHEMATICAL AREAS AND GESTALTS

After we developed the Logical Math and Computational Math scales, we used these scales to measure the preferred gestalt in different areas of mathematics. We classified each of our 112 sample books into a Traditional Math area (Algebra, Analysis, Geometry, etc.) or an Applied Math area (Differential Equations, Numerical Analysis, Operations Research, etc.). Traditional Math areas are fairly well defined, whereas Applied Math areas are not universally agreed upon.

From the books in each Math area, we calculated the average LMATH score and average CMATH score. We also counted the number of books having an LMATH or CMATH score above 200. No book scored above 200 on both scales, indicating that these books favor at most one mathematical gestalt.

Math areas were then grouped and sorted according to which average scale score was higher. Higher average LMATH value means Logical Math gestalt is emphasized. Higher average CMATH value indicates Computational Math gestalt is favored. The results are summarized in Appendix 2.

The Math areas of Logic, Topology, Analysis, and Number Theory have LMATH averages above 275, plus all of these books have individual LMATH scores above 200. Other Math areas with average LMATH above 200 include Probability, Calculus, and Algebra. The majority of the books in these areas have LMATH scores above 200. Surprisingly, the LMATH average for Geometry is relatively low at 146.4, but only 2 Geometry books were in the sample. We note that two key words in Geometry are "point" and "line", but these words are not on the LMATH scale.

The LMATH averages for Probability and Algebra would have been higher, if the books in these areas were more uniform. Several Probability books include applications, but most are theoretical and treat Probability as a subfield of Analysis (Measure Theory). The Algebra area includes 4 Abstract Algebra books and 3 Linear Algebra books. The LMATH average for the Abstract Algebra books is 289.6, with individual values ranging from 235.8 to 358.4. The Linear Algebra books have a CMATH average of 165.3, with one individual value above 200.

The Math areas with CMATH averages above 250 are Operations Research, Optimization, and Numerical Analysis. All 5 Operations Research books have a CMATH score above 200. The remaining Math areas with average CMATH values above 200 are Applied Math, Differential Equations, Math Modeling, and Computational Math.

The CMATH scores within Applied Math areas vary widely, since these books cover a broad range of subjects. For example, Differential Equations books include Ordinary Differential Equations and Partial Differential Equations. Some of the Differential Equation books are theoretical, and others are computational. The average LMATH score for the two theoretical Partial Differential Equations books is 294.6. The CMATH average for the other four books is 270.2.

Operations Research books are the prototype for problem-solving gestalt, since these books frequently use all of the words "problem", "model", "method/algorithm" and "solution/solve". Numerical Analysis and Differential Equation books tend not to use the word "model". Simulation emphasizes sampling the behavior of models instead of "solving" them. The field of Statistics uses models and methods, but prefers estimates and tests over solutions. Nevertheless, we did find one Simulation book with a CMATH score above 200, one Statistics book with a CMATH score above 200, and one Statistics book with an LMATH score above 200.

6. SUMMARY AND CONCLUSIONS

The purpose of this study was to develop measuring instruments for two mental frameworks, or gestalts, in mathematics. Logical Math gestalt is based on proving theorems, while Computational Math gestalt is based on solving problems. From a non-random but diverse sample of 56 Traditional Math books and 56 Applied Math books, we examined the 100 most frequently used words in each book. Weighted combinations of selected words were used to form a Logical Math (LMATH) scale and a Computational Math (CMATH) scale.

Our LMATH scale contains 10 words/groups, including *theorem/lemma*, *proof*, *let*, and *definition*. The CMATH scale has 9 words/groups, including *problem*, *solution*, *model*, and *method/algorithm*. Books with high LMATH scores represent the mathematical areas of Logic, Topology, Analysis, Number Theory, Probability, Calculus, and Algebra. Books with high CMATH scores come from the areas of Operations Research, Optimization, Numerical Analysis, Applied Math, Differential Equations, Math Modeling, and Computational Math.

Suggestions for Future Research

As part of our continuing research, we have applied the LMATH and CMATH scales to a sample of Discrete Mathematics textbooks. Our intent is to determine which type of gestalt is emphasized in these books, and to show how this will influence the way Discrete Mathematics courses are taught.

We are applying the LMATH and CMATH scales to other text materials, such as journal articles and research papers. For example, we calculated LMATH and CMATH scores for Wing's (2006) article on "Computational Thinking". The LMATH score of 14.9 indicates virtually no Logical Math gestalt. The CMATH score was 144.5, which suggests a modest level of Computational Math gestalt. However, if the word "computation" (which is not on our CMATH scale) were treated as a synonym for "algorithm", then Wing's CMATH score would rise to 256.2.

Logical Math and Computational Math are not the only mental frameworks relevant to Information Systems. We are continuing our efforts to combine mathematical and software gestalts in computing courses. A previous paper (McMaster, Anderson, Rague, 2007) described how to integrate programming into the Discrete Mathematics course. We are now working to define scales for measuring gestalts in software development. We then plan to relate software development frameworks to the two mathematical gestalts described in this paper.

Our goal is to establish ways to successfully blend mathematical concepts with software development. For example, in Computational Math gestalt, one of the scale words is "solution." Analysis of a preliminary sample of software engineering books indicates that the most frequent word is "software."

The field of Information Systems differs from Mathematics in that our solutions are implemented as software. Problem-solving gestalt in software development might well be described by the sequence:

Problem-->Model-->Algorithm-->Software

This scheme is remarkably similar to the suggestion by Zachary (1997) that the gestalt of scientific programming involves problem, model, method, and implementation. For an information system, software (implementation) is the solution.

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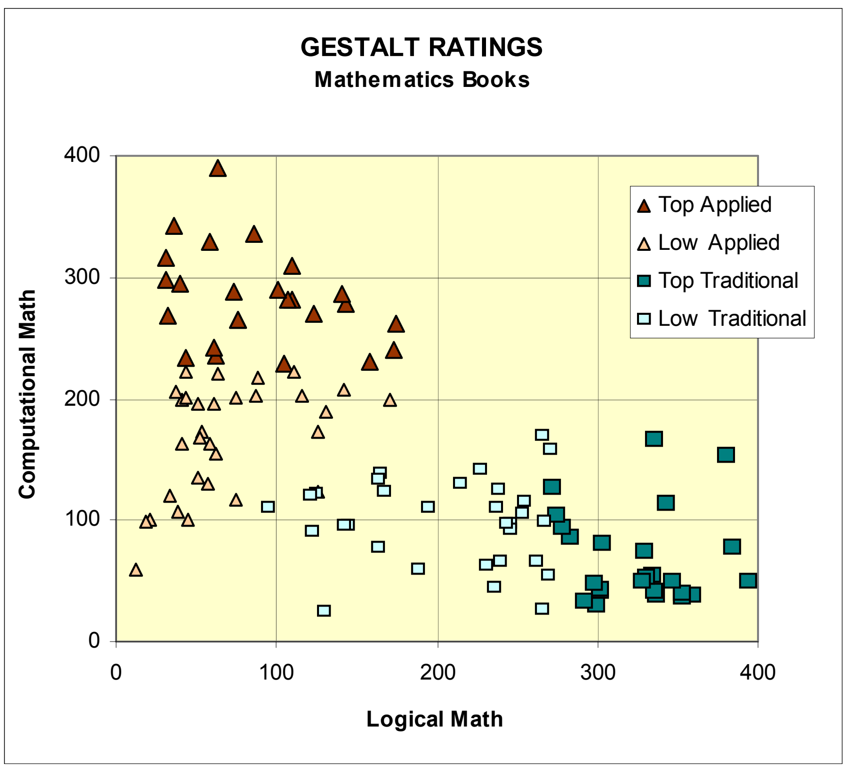
APPENDIX 1: LOGICAL MATH VS. COMPUTATIONAL MATH

All mathematics books in the sample



APPENDIX 2: LMATH AND CMATH SCALES AND MATHEMATICAL AREA

Mathematical Area	Avg LMATH	Avg CMATH	LMATH > 200	CMATH > 200	Total Books
Logical Math Gestalt					
Mathematical Logic	**323.6**	52.0	6		6
Topology	**311.1**	51.0	4		4
Analysis	**302.6**	81.5	8		8
Number Theory	**279.7**	89.0	6		6
Probability	**264.6**	91.6	6		8
Calculus	**227.4**	123.6	3		4
Algebra	**204.1**	90.4	4	1	7
Geometry	146.4	51.2			2
Computational Math Gestalt					
Operations Research	38.8	**292.9**		5	5
Optimization	152.3	**274.8**	1	3	4
Numerical Analysis	91.7	**256.3**		2	3
Applied Math	115.7	**227.7**		6	9
Differential Equations	169.9	**227.2**	2	3	6
Math Modeling	47.3	**226.2**		4	6
Computational Math	114.6	**212.3**	1	8	11
Simulation	44.9	146.1		1	6
Statistics	102.6	120.7	1	1	9
Graphics	48.2	117.7			2
Miscellaneous	140.4	132.0	1	1	6
Total	--	--	43	35	112